Rice Oil & Gas HPC Conference — March 04th–06th, 2019, Houston, Texas, USA

Multiphase Fluid Flow Simulator with Applications in Porous Media

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Abstract. Two-phase flow with viscosity contrast at the pore scale is modeled by a time-dependent Cahn–Hilliard–Navier–Stokes model and belongs to the class of diffuse interface method. The model allows for moving contact line and varying wettability. The numerical scheme utilizes an efficient pressure-correction projection algorithm, in conjunction with interior penalty discontinuous Galerkin (DG) schemes for space discretization developed within the framework of a distributed parallel pore-scale flow simulation system. The effect of viscosity contrast on the phase distribution is studied in relation with capillary forces and wettability. The algorithm is numerically robust and lends itself naturally to large-scale 3D numerical simulations.

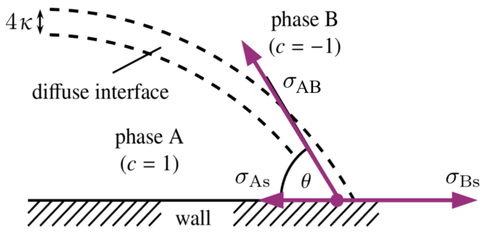
Phase-field Model

Cahn–Hilliard–Navier–Stokes System

- The unknown variables are **order parameter** c, **chemical potential** μ, **velocity** $\boldsymbol{v}$, and **pressure** p.

$$\begin{aligned}
\partial_t c - \boldsymbol{\nabla}\cdot(M(c)\,\boldsymbol{\nabla}\mu) + \boldsymbol{\nabla}\cdot(c\boldsymbol{v}) &= 0 \text{ in } (0,T)\times\Omega,\\
\mu = \beta\,\Phi'(c) - \alpha\,\Delta c &\text{ in } (0,T)\times\Omega,\\
\rho_0(\partial_t \boldsymbol{v} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\boldsymbol{v}) - 2\boldsymbol{\nabla}\cdot(\mu_s\boldsymbol{\varepsilon}(\boldsymbol{v})) &= -\boldsymbol{\nabla}p + \mu\boldsymbol{\nabla}c \text{ in } (0,T)\times\Omega,\\
\boldsymbol{\nabla}\cdot\boldsymbol{v} &= 0 \text{ in } (0,T)\times\Omega.
\end{aligned}$$

- For flow through porous media scenarios, the model is supplemented with physically and mathematically **consistent boundary conditions** at the inlet and outlet.
- The relationship between **contact angle** and **surface tensions** is given by **Young's equation**.



- Total energy F equals kinetic energy plus Helmholtz free energy plus surface energy:

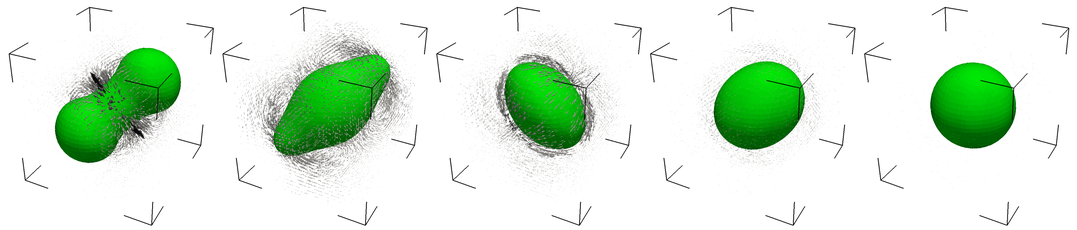
$$F(c,\boldsymbol{v}) = \int_\Omega \frac{\rho_0}{2}|\boldsymbol{v}|^2 + \int_\Omega \left(\beta\,\Phi(c) + \frac{\alpha}{2}|\boldsymbol{\nabla}c|^2\right) + \int_{\partial\Omega}\Big((\sigma_{\mathrm{Bs}} - \sigma_{\mathrm{As}})g(c) + \sigma_{\mathrm{As}}\Big),$$

where $\Phi(c)$ denotes the **chemical energy density** and $g(c)$ denotes the **blending function**.

- Popular expressions of Φ include **Ginzburg–Landau potential** and **Flory–Huggins potential**.
- The relationship between shear viscosity and order parameters is given by the **mixing rule**: linear rule, harmonic rule, or exponential rule.
- A closed Cahn–Hilliard–Navier–Stokes system enjoys the properties of **mass conservation** and **energy dissipation**.

[1] F. Frank, C. Liu, F. O. Alpak, S. Berg, and B. Rivière. "Direct numerical simulation of flow on pore-scale images using the phase-field method". In: *SPE Journal*, 23(5) (2018), pp. 1833–1850.

Merging Droplets



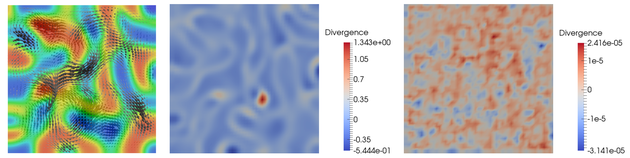
3D views of the evolution of the **order parameter** c (interface is shown in green and phase B is transparent) and the **velocity field** $\boldsymbol{v}$. The two droplets are merging together until a stationary state is reached.

Two-phase Flow through Porous Domains

Micro-CT scan creates **porous images** at micrometer scale in which **voxel sets** represent the structure of porous media. The figure shows the order parameter field (red for phase A, green for the interface center, and blue / transparent for phase B).

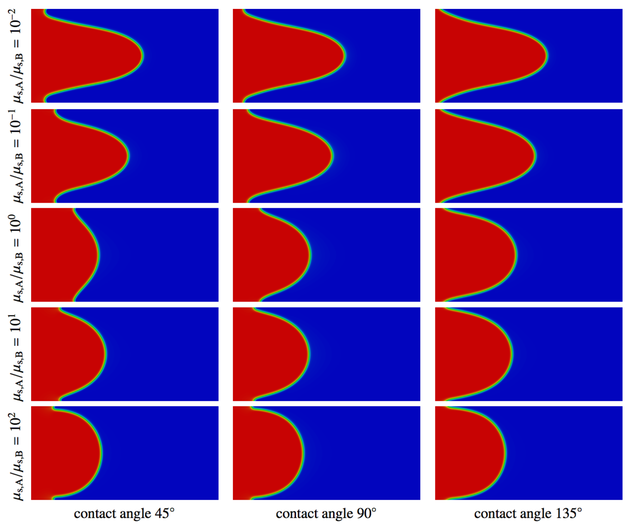
Numerical Method

- **Hierarchical bases** with orthogonal basis functions enable arbitrary order of approximation.
- **Interior penalty DG** methods for space discretization.
- **Implicit-explicit** scheme for time discretization.
- Rotational incremental **Pressure-correction projection** algorithm in conjunction with **div–div correction technique** ensures a pointwise solenoidal velocity field.



- Momentum balance equation: linearized by **Picard splitting**.
- Mass balance equation: linearized by **Newton's method**. **Dissipates discrete free energy** by utilizing a **convex–concave decomposition**. Scheme reduces to **cell-centered finite volumes** with the use of element-wise constants basis [1].

Cylindrical Pipe Simulations



Two-dimensional views of two-phase distribution in a cylinder.